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Short Communication

Multi-input robust saturation controller for uncertain linear time-invariant systems

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Abstract

A robust saturation controller for the linear time-invariant (LTI) system involving both a control input's saturation and structured real parameter uncertainties was proposed in [C.W. Lim, Y. J. Park, S.J. Moon, Robust saturation controller for linear time-invariant system with structured real parameter uncertainties, *Journal of Sound and Vibration* 294 (1–2) (2006) 1–14]. This controller can also be applicable to the multi-input case. In this paper, the robust saturation controller is extended to the uncertain LTI system with multi-input and designed by introducing additional subsidiary setting parameters for each control input. An example is presented to show its application to multi-input uncertain LTI systems.

1. Introduction

The control of a dynamic system with bounded control input has been one of the basic problems in many control engineering fields such as mechanical engineering, aerospace engineering, civil engineering, chemical engineering, and so on. An extensive chronological bibliography on the progress of dealing with control input's saturation problem has been presented in Ref. [1] and some fundamental problems of control systems with control input's saturation have been examined in a systematic manner in Ref. [2]. Since the traditional linear controllers are designed to be stable without control input's saturation, the stability of the closed-loop system is not guaranteed and the closed-loop system can become unstable when the control input is saturated [3,4].

One can avoid control input's saturation by imposing the constraint on it so that the required control input may be smaller than the bounded control input. But the control performance may be seriously compromised in some cases due to the use of low-gain. Another case to deal with the saturation problem is the bang–bang controller which can fully utilize the bounded control input in the design. The stability of the closed-loop system during control input's saturation for the bang–bang controller is guaranteed. There have been several

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types of bang–bang controllers for nominal linear time-invariant (LTI) systems like optimal bang–bang controller, suboptimal bang–bang controller, modified bang–bang controller, and so on [5–10]. Among them, the modified bang–bang controller is most physically applicable to real systems.

To guarantee the robust stability of the bang–bang type controllers, a single input robust saturation controller for the LTI system involving both a control input's saturation and structured real parameter uncertainties was proposed in [11]. The robust saturation controller is an extension of the modified bang–bang controller for nominal LTI systems into uncertain LTI systems. In this paper, a multi-input robust saturation controller is proposed for the uncertain LTI systems with multi-input. A design scheme of the proposed controller is presented and its application example is shown.

2. Multi-input robust saturation controller

Here, the following uncertain LTI system with multi-input (1) is considered and a robust saturation controller for the multi-input case is designed based on the affine quadratic stability definition and multi-convexity concept [12] which are analytical tools for the design of this controller:

$$\dot{x}(t) = (A_0 + \Delta A(\theta))x(t) + BU(t), \quad x(0) = x_0, \tag{1}$$

where $x = [x_1\, x_2 \ldots x_n]^{\mathrm{T}}$ is an $n \times 1$ state-vector, A_0 is an $n \times n$ nominal system matrix, $\theta = (\theta_1, \theta_2, \ldots, \theta_k) \in \Re^k$ is a vector of uncertain real parameters, $\Delta A(\theta)$ is time-invariant uncertainties, $A_0 + \Delta A(\theta)$ is assumed to be stable, B is an $n \times r$ control input matrix with the jth element B_j, and $U(t)$ is an $r \times 1$ vector consisting of r control inputs. And $u_j(t)$ is the jth control input and is bounded by $\pm u_{j\max}$ as Eq. (2).

$$|u_j(t)| \leqslant u_{j\,\max} \quad \text{for} \quad j = 1, 2, \ldots, r. \tag{2}$$

We assume that lower and upper bounds are available for the parameter values. Specifically, each parameter θ_i ranges between known external values $\underline{\theta_i}$ and $\overline{\theta_i}$.

$$\theta_i \in [\underline{\theta_i}, \overline{\theta_i}] \quad \text{for} \quad i = 1, 2, \ldots, k. \tag{3}$$

This means that the parameter vector θ is valued in a hyper-rectangle called the parameter box. In the sequel

$$\Theta := \{(\omega_1, \omega_2, \ldots, \omega_k) : \omega_i \in \{\underline{\theta_i}, \overline{\theta_i}\}\} \tag{4}$$

denotes the set of the 2^k vertices or corners of this parameters.

The uncertain system matrix $A(\theta)$ depends affinely on the uncertain parameters of θ_i and is described by the system with structured real parameter uncertainties. That is

$$A(\theta) = A_0 + \Delta A(\theta) = A_0 + \theta_1 A_1 + \theta_2 A_2 + \cdots + \theta_k A_k, \tag{5}$$

where A_0, A_1, A_2,… A_k are known fixed matrices.

The following notion of parameter-dependent quadratic Lyapunov function is defined.

$$V(x(t), \theta) = x^{\mathrm{T}} P(\theta) x, \tag{6}$$

where $P(\theta)$ is an affine function of θ and is composed of a positive-definite symmetric matrix P_0 and k symmetric matrices P_1, P_2,…,P_k.

$$P(\theta) = P_0 + \Delta P(\theta) = P_0 + \theta_1 P_1 + \theta_2 P_2 + \cdots + \theta_k P_k. \tag{7}$$

The time derivative of the Lyapunov function (6) is of the following form:

$$\dot{V}(x(t), \theta) = x^{\mathrm{T}}(t)[A(\theta)^{\mathrm{T}} P(\theta) + P(\theta)A(\theta)]x(t) + 2x^{\mathrm{T}}(t)P(\theta)BU(t). \tag{8}$$

For $\dot{V}(x(t), \theta) < 0$ with control input's constraint of Eq. (2), the following multi-input robust saturation controller of saturation function type are proposed.

$$U(t) = \mathrm{diag}(u_1(t),\; u_2(t), \ldots, u_r(t)) \quad \text{or} \quad u_j(t) = -\mathrm{sat}[\delta_j B_j^{\mathrm{T}} P_0 x(t)], \tag{9}$$

where $\delta_j > 0 (j = 1, 2, \ldots, r)$ and P_0 satisfies $2^{k+1} + k$ LMI conditions of Eqs. (10)–(12).

The following Theorem 1 gives sufficient conditions for the existence of the robust saturation controller (9). LMIs of Eqs. (10)–(12) can be easily solved using commercial MATLAB® and LMI control toolbox [13], which is very efficient and easy to use, by setting M_a in an arbitrary positive-definite symmetric matrix. M_a is a controller design parameter and positive-definite symmetric matrices $M_{aj}(j = 1, 2, \ldots, r)$, which satisfy Eq. (13) and are not necessary in the single input case, are additional subsidiary setting parameters for each control input. Setting each matrix of M_{aj} determines the maximum magnitude of the jth control input.

Theorem 1. Consider an uncertain linear time-invariant system (1) where $A(\theta)$ depends affinely on the parameter vector $\theta = (\theta_1, \theta_2, \ldots, \theta_k)$, θ_i satisfies Eq. (3), and control inputs have constraint of Eq. (2). Let Θ denotes the sets of vertices of the parameter box Eq. (4). Robust stability of the multi-input robust saturation controller (9) is guaranteed if there exist $k+1$ symmetric matrices P_0, P_1, $P_2,\ldots$, P_k, and a positive-definite symmetric matrix M_a satisfying Eqs. (10)–(12), if there exist positive-definite symmetric matrices M_{aj} $(j = 1, 2, \ldots, r)$ satisfying Eq. (13), and if there exist $\delta_j > 0$ $(j = 1, 2, \ldots, r)$ which satisfy Eq. (14) for these matrices P_0, P_1, $P_2,\ldots$, P_k and M_{aj}.

$$P(\omega) > 0 \quad \text{for all } \omega \in \Theta, \tag{10}$$

$$A(\omega)^\mathrm{T} P(\omega) + P(\omega)A(\omega) + M_a < 0 \quad \text{for all } \omega \in \Theta, \tag{11}$$

$$A_i^\mathrm{T} P_i + P_i A_i \geqslant 0 \quad \text{for } i = 1, 2, \ldots, k, \tag{12}$$

$$\sum_{j=1}^{r} M_{aj} = M_a, \tag{13}$$

$$M_{aj} + \delta_j \left\{ 2P_0 B_j B_j^\mathrm{T} P_0 + \sum_{i=1}^{k} \theta_i (P_0 B_j B_j^\mathrm{T} P_i + P_i B_j B_j^\mathrm{T} P_0) \right\} > 0 \quad \text{for all } \omega \in \Theta \text{ and for } j = 1, 2, \ldots, r. \tag{14}$$

Proof. We can express control inputs (9) as the following form by introducing $\beta_j(x(t))$.

$$\begin{aligned} u_j(t) &= -\beta_j(x(t))\delta_j B_j^\mathrm{T} P_0 x(t), \\ \beta_j(x(t)) &= \frac{\mathrm{sat}(\delta_j B_j^\mathrm{T} P_0 x(t))}{\delta_j B_j^\mathrm{T} P_0 x(t)}, \\ \beta_j(x(t)) &= 1 \quad \text{if} \quad B_j^\mathrm{T} P_0 x(t) = 0, \end{aligned} \tag{15}$$

where $0 < \beta_j(x(t)) \leqslant 1$ for $j = 1, 2, \ldots, r$.

Along the trajectories of system (1) with the control inputs given in Eq. (9), the time derivative of $V(x, \theta)$ is obtained as

$$\begin{aligned} \dot{V}(x,\theta) &= x^\mathrm{T}[A(\theta)^\mathrm{T} P(\theta) + P(\theta)A(\theta)]x \\ &\quad + x^\mathrm{T} \left[\sum_{j=1}^{r} -\beta_j \delta_j \left(2P_0 B_j B_j^\mathrm{T} P_0 + \sum_{i=1}^{k} \theta_i (P_0 B_j B_j^\mathrm{T} P_i + P_i B_j B_j^\mathrm{T} P_0) \right) \right] x. \end{aligned} \tag{16}$$

Because it is not guaranteed that the second term in the right-hand term of Eq. (16) is less than 0 with $\delta_j > 0$, a positive-definite symmetric matrix M_a is introduced to make $\delta_j > 0$. Eq. (17) is obtained from adding and subtracting M_a each term in the right-hand term of Eq. (16).

$$\begin{aligned} \dot{V}(x,\theta) &= x^\mathrm{T}[A(\theta)^\mathrm{T} P(\theta) + P(\theta)A(\theta) + M_a]x \\ &\quad + x^\mathrm{T} \left[-M_a - \sum_{j=1}^{r} \beta_j \delta_j \left(2P_0 B_j B_j^\mathrm{T} P_0 + \sum_{i=1}^{k} \theta_i (P_0 B_j B_j^\mathrm{T} P_i + P_i B_j B_j^\mathrm{T} P_0) \right) \right] x. \end{aligned} \tag{17}$$

We assume that there exist symmetric matrices P_0, P_1, $P_2,\ldots$, P_k, and M_a satisfying that the first term in the right-hand term of Eq. (17) is less than 0. Robust stability of the multi-input robust saturation controller (9) is guaranteed if the second term in the right-hand term of Eq. (17) is less than 0 with $\delta_j > 0$ when we substitute these matrices $P_0, P_1, P_2, \ldots, P_k$, and M_a into the second term in the right-hand term of Eq. (17). The first term in the right-hand term of Eq. (17) is always less than 0 if there exist symmetric matrices $P_0, P_1, P_2, \ldots, P_k$, and a positive-definite symmetric matrix M_a satisfying Eqs. (10)–(12). Let $\rho_{ji} = \delta_j\theta_i$ ($j = 1, 2, \ldots, r$ and $i = 1, 2, \ldots, k$) and $M_a = \sum_{j=1}^{r} M_{aj}$ like Eq. (13), then $\delta_j \underline{\theta_i} \leqslant \rho_{ji} \leqslant \delta_j \overline{\theta_i}$ and the second term in the right-hand term of Eq. (17) is rewritten as Eq. (18).

$$-x^{\mathrm{T}}\left[\sum_{j=1}^{r}\left\{M_{aj} + \beta_j\left(\delta_j(2P_0B_jB_j^{\mathrm{T}}P_0) + \sum_{i=1}^{k}\rho_{ji}(P_0B_jB_j^{\mathrm{T}}P_i + P_iB_jB_j^{\mathrm{T}}P_0)\right)\right\}\right]x. \tag{18}$$

Here, we first consider the case of $\beta_j = 1$ for $j = 1, 2, \ldots, r$. For given $\delta_j > 0$, all the following LMIs of Eq. (19) are a convex constraint on the variables ρ_{ji} because M_{aj}, $P_0B_jB_j^{\mathrm{T}}P_0$, and $P_0B_jB_j^{\mathrm{T}}P_i + P_iB_jB_j^{\mathrm{T}}P_0$ are symmetric matrices, respectively.

$$M_{aj} + \delta_j(2P_0B_jB_j^{\mathrm{T}}P_0) + \sum_{i=1}^{k}\rho_{ji}(P_0B_jB_j^{\mathrm{T}}P_i + P_iB_jB_j^{\mathrm{T}}P_0) > 0 \quad \text{for } j = 1, 2, \ldots, r. \tag{19}$$

When we define Φ_j ($j = 1, 2, \ldots, r$) as the set of the 2^k vertices of ρ_{ji} of Eq. (20), Eq. (19) is satisfied for all ρ_{ji} if and only if Eq. (19) is satisfied in Φ_j by convexity of Eq. (19).

$$\Phi_j := \{(\psi_{j1}, \psi_{j2}, \ldots, \psi_{jk}) : \psi_{ji} \in \{\delta_j\underline{\theta_i}, \delta_j\overline{\theta_i}\}\} \quad \text{for } j = 1, 2, \ldots, r. \tag{20}$$

Eq. (19) is equivalent to Eq. (14). Next, we consider the case of $0 < \beta_j < 1$ for $j = 1, 2, \ldots, r$. For given $\delta_j > 0$, we can easily show that Eq. (18) is less than 0 if Eq. (19) is satisfied. Therefore Eq. (18) is always less than 0 if Eq. (14) is satisfied for given $\delta_j > 0$.

Remark 1. A numerical method to overcome the difficulty of the multi-convexity constraint of Eq. (12) was suggested in Refs. [11,12]. M_a and $P_0, P_1, P_2, \ldots, P_k$ satisfying the LMIs of Eqs. (10)–(12) can be sought simultaneously. But it is more practical and desirable to let M_a be a controller design parameter and set it in an arbitrary value by controller designer. Then $P_0, P_1, P_2, \ldots, P_k$ can be obtained by solving the LMIs of Eqs. (10)–(12) given M_a. In the LMI control toolbox of MATLAB® [13], the solution of $P_0, P_1, P_2, \ldots, P_k$ to simultaneous LMIs of Eqs. (10)–(12) is formulated as

$$\text{Minimize } \tau \text{ subject to } \mathrm{LH}(X) < \mathrm{RH}(X) + \tau I, \tag{21}$$

where LH(X) and RH(X) are left-hand and right-hand sides of LMIs respectively. Both LH(X) and RH(X) are function of the matrix variable X and I is an identity matrix with an appropriate dimensions. The condition for the existence of a solution to LMIs is obtained from the global minimum of τ, denoted by $\tau_{\min}$. If $\tau_{\min} < 0$, there exists a solution of the LMIs, whereas $\tau_{\min} \geqslant 0$ means that the solution does not exist.

Remark 2. A simple method to determine subsidiary setting parameters M_{aj} in Eq. (13) is to choose $M_{aj} = \alpha_j M_a (\sum_{j=1}^{r}\alpha_j = 1)$ with $\alpha_j < 0$. And all the maximum values of δ_j for the jth control input satisfying Eq. (14) are finite ($0 < \delta_j \leqslant \delta_{j\,\max}$). We can seek $\delta_{j\max}$ by setting δ_j in a fixed value and sweeping through δ_j in Eq. (14). The larger the bounds of parameter uncertainties are, the smaller the maximum values of δ_j are in general.

3. Numerical simulations

The proposed multi-input robust saturation controller (9) is applied to a 2DOF linear vibrating system with two control input forces as shown in Fig. 1. The masses, stiffnesses, and damping coefficients for nominal system are $m_1 = m_2 = 1\,\mathrm{kg}$, $k_1 = k_2 = 1\,\mathrm{N/m}$, and $c_1 = c_2 = 0.01\,\mathrm{Ns/m}$, respectively. The maximum control input forces are $u_{1\,\max} = 0.1\,\mathrm{N}$ and $u_{2\,\max} = 0.1\,\mathrm{N}$. Let uncertainties of stiffnesses be θ_1 and θ_2, then the admissible trajectories are given by $k_1(1+\theta_1)$ and $k_2(1+\theta_2)$ specified in multiplicative form and this uncertain system can be described by state space equation as in Eq. (1). In this case, state vector $x = [x_1 \quad x_2 \quad \dot{x}_1 \quad \dot{x}_2]^{\mathrm{T}}$, control input matrix $B = [B_1 \quad B_2]$ ($B_1 = [0 \quad 0 \quad 1/m_1 \quad 0]^{\mathrm{T}}$ and $B_2 = [0 \quad 0 \quad 0 \quad 1/m_2]^{\mathrm{T}}$), and uncertain

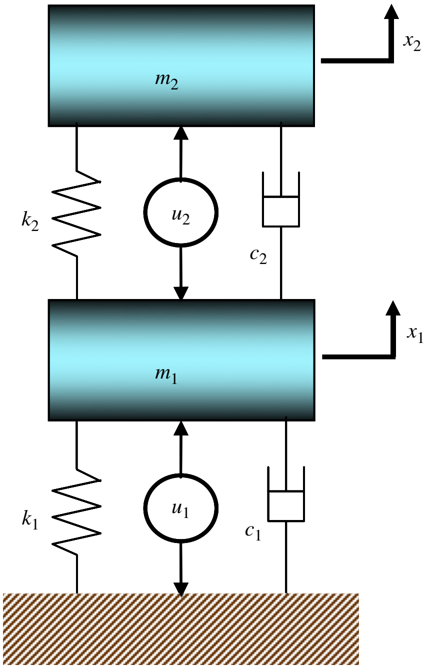

Fig. 1. 2DOF linear vibrating system with two control input forces.

Table 1
Obtained values of P_0, P_1 and P_2

P_0	$\begin{bmatrix} 2.5982e+1 & -1.2982e+1 & 1.1143e-1 & -5.0910e-2 \\ -1.2982e+1 & 1.2998e+1 & -5.3299e-2 & 6.1955e-2 \\ 1.1143e-1 & -5.3299e-2 & 1.3003e+1 & 1.5260e-2 \\ -5.0910e-2 & 6.1955e-2 & 1.5260e-2 & 1.3016e+1 \end{bmatrix}$
P_1	$\begin{bmatrix} 1.2992e+1 & 6.1831e-3 & -3.2421e-2 & -4.5517e-3 \\ 6.1831e-3 & 6.9763e-3 & 4.6074e-3 & 6.4153e-4 \\ -3.2421e-2 & 4.6074e-3 & 2.3088e-4 & -2.3677e-4 \\ -4.5517e-3 & 6.4153e-4 & -2.3677e-4 & 6.4012e-3 \end{bmatrix}$
P_2	$\begin{bmatrix} 1.2984e+1 & -1.2987e+1 & -3.1495e-2 & 4.1427e-2 \\ -1.2987e+1 & 1.2992e+1 & 2.3830e-2 & -3.6864e-2 \\ -3.1495e-2 & 2.3830e-2 & -1.6347e-3 & -1.7168e-3 \\ 4.1427e-2 & -3.6864e-2 & -1.7168e-3 & -1.7614e-3 \end{bmatrix}$

system matrix $A(\theta)$ is described by

$$A(\theta) = A_0 + \theta_1 A_1 + \theta_2 A_2, \tag{22}$$

where

$$A_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{k_1+k_2}{m_1} & \dfrac{k_2}{m_1} & -\dfrac{c_1+c_2}{m_1} & \dfrac{c_2}{m_1} \\ \dfrac{k_2}{m_2} & -\dfrac{k_2}{m_2} & \dfrac{c_2}{m_2} & -\dfrac{c_2}{m_2} \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\dfrac{k_1}{m_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\dfrac{k_2}{m_1} & \dfrac{k_2}{m_1} & 0 & 0 \\ \dfrac{k_2}{m_2} & -\dfrac{k_2}{m_2} & 0 & 0 \end{bmatrix}.$$

Simulation results for the case with parameter uncertainties of $|\theta_1| \leqslant 0.5$ and $|\theta_2| \leqslant 0.5$ are presented under the initial condition of $x_0 = [0 \quad 0 \quad -0.5 \quad 1.5]^T$. The controller design parameter $M_a = \mu_a I$ with $\mu_a = 1e-4$ is chosen. The obtained values of P_0, P_1 and P_2 are shown in Table 1 with $\tau_{min} = -4.0911e-4$. And additional subsidiary setting parameters for each control input are let as $M_{a1} = \alpha_1 M_a$ with $\alpha_1 = 0.6$ and $M_{a2} = \alpha_2 M_a$ with $\alpha_2 = 0.4$. The computed maximum values of δ_1 and δ_2 are $\delta_{1\,max} = 9.80e-2$ and $\delta_{2\,max} = 9.09e-2$. The robust saturation controller with $\delta_1 = \delta_{1\,max}$ and $\delta_2 = \delta_{2\,max}$ was used in numerical simulations.

Fig. 2 shows displacements and control input forces for nominal system applying the multi-input robust saturation controller. The control performance is considerably effective. The multi-input robust saturation

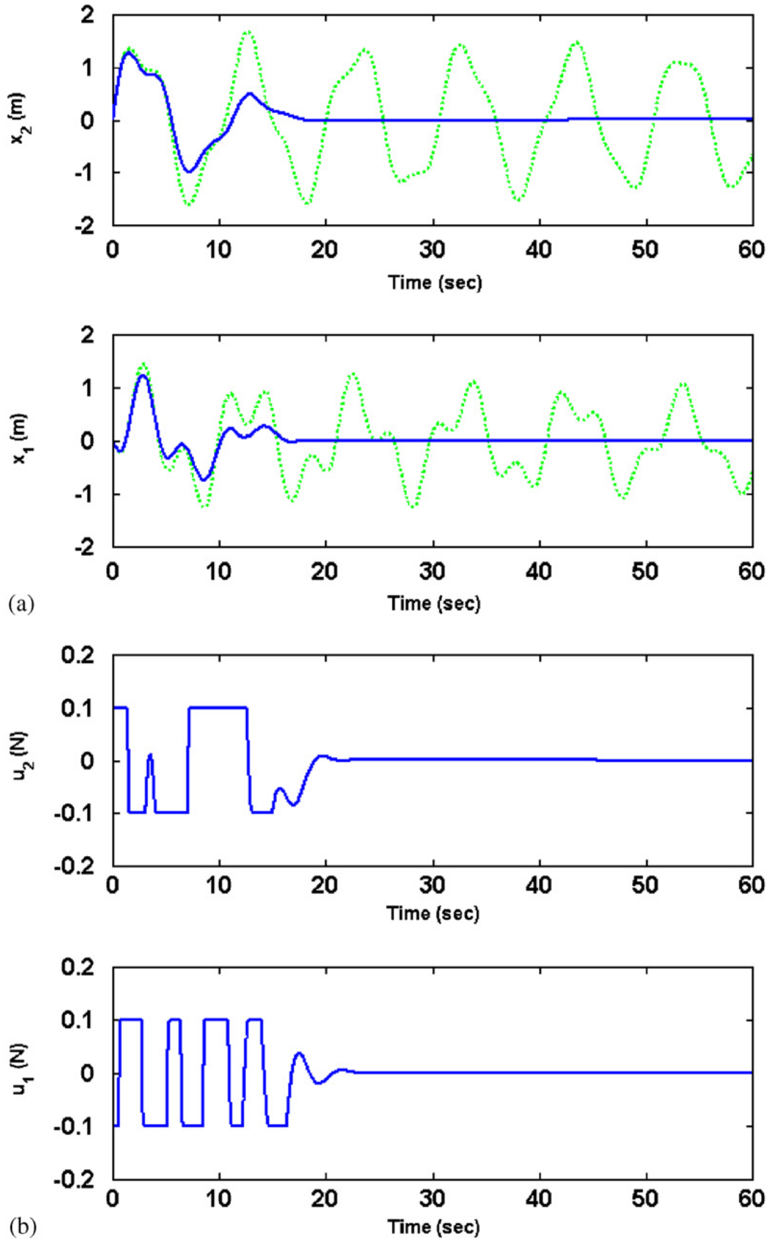

Fig. 2. Displacements and control input forces for nominal system applying the multi-input robust saturation controller (-−-, No control; —, Controlled). (a) displacements, (b) control input forces.

controller (9) guarantees robust stability analytically within all the range of parameter uncertainties considered in controller design. Its robust stability can be also checked through numerical simulations. Among various cases, Figs. 3 and 4 show the results for the uncertain system with $\theta_1 = \theta_2 = 0.5$ and for the uncertain system with $\theta_1 = \theta_2 = -0.5$ applying the multi-input robust saturation controller, respectively. Simulation results show that the multi-input robust saturation controller is robustly stable with respect to parameter uncertainties over the prescribed lower and upper bounds and can be effectively applied to the uncertain LTI systems with multi-input.

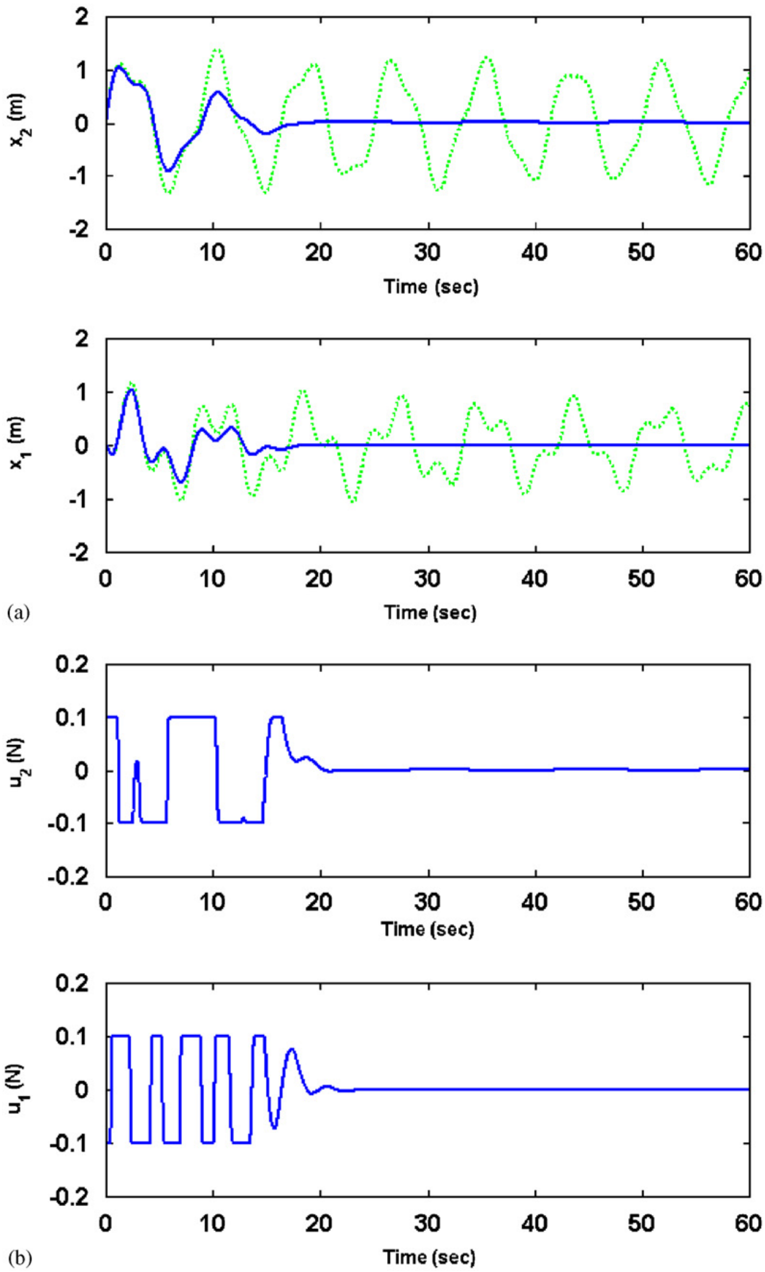

Fig. 3. Displacements and control input forces for uncertain system with $\theta_1 = \theta_2 = 0.5$ applying the multi-input robust saturation controller (−−, No control; —, Controlled). (a) displacements, (b) control input forces.

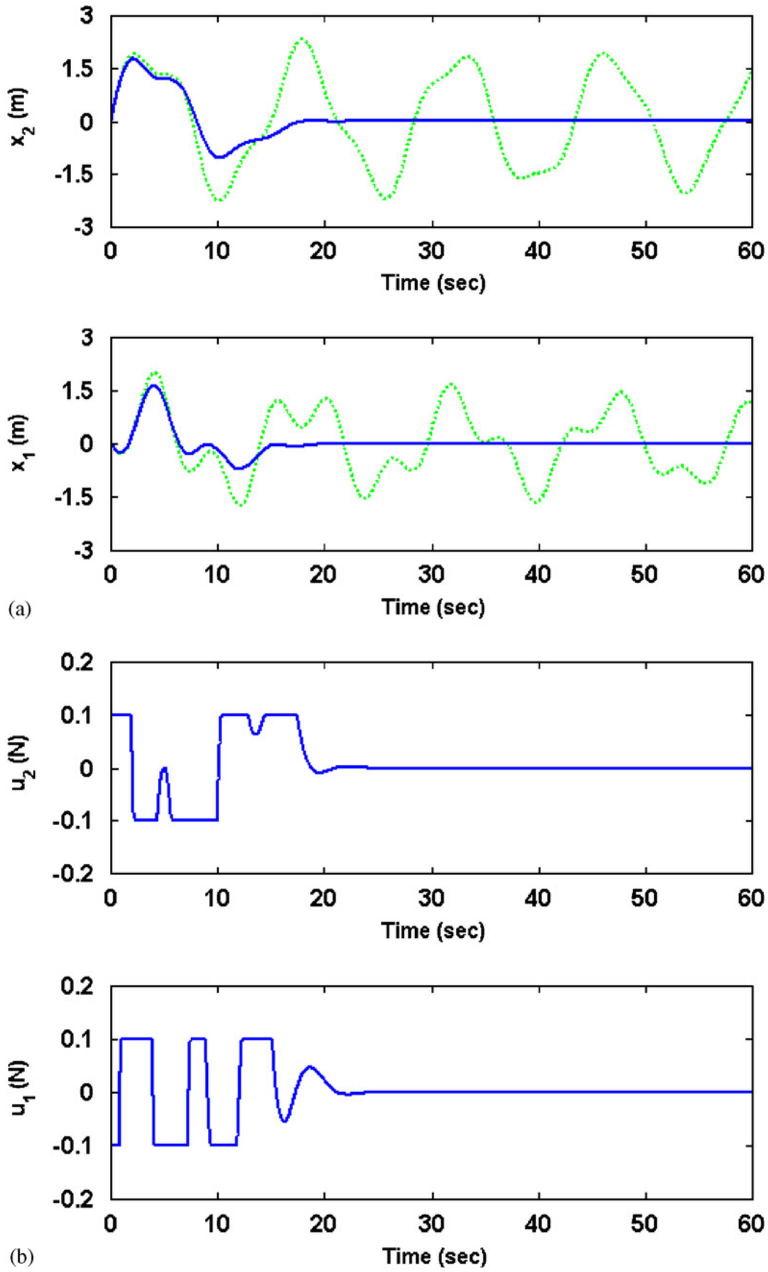

Fig. 4. Displacements and control input forces for uncertain system with $\theta_1 = \theta_2 = -0.5$ applying the multi-input robust saturation controller (––, No control; —, Controlled). (a) displacements, (b) control input forces.

4. Conclusions

In this paper, a multi-input robust saturation controller for uncertain LTI systems was proposed. The controller was designed by introducing additional subsidiary setting parameters M_{aj} $(j = 1, 2, \ldots, r)$, which are not necessary in the single input case, for each control input. It was shown through numerical simulations that the multi-input robust saturation controller is effectively applicable to the multi-input uncertain LTI systems involving both control inputs' saturation and structured real parameter uncertainties.

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